$$= \sum_{m=1}^{n} C_m(t).$$

The cost of machine in 'n' year.

$$= C + \sum_{m=1}^{n} C_m(t) - S.$$

Average cost of machine per year during

$$\boxed{A(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{m=1}^{n} C_m(t)\ \cancel{dt}} \quad \text{—— (1)}$$

For min.

$$\left.\begin{array}{l} \Delta A(n-1) < 0. \\ \text{and} \quad \Delta A(n) > 0. \end{array}\right] \text{—— (a)}$$

Where Δ is forward difference.

$$\Delta f(x) = f(x+h) - f(x) \quad \text{if } h=1.$$
$$= f(x+1) - f(x).$$
$$\Delta A(n) = A(n-1) - A(n).$$
$$\Delta A(n) = \left[\frac{(C-S)}{n+1} - \frac{1}{(n+1)}\sum_{m=1}^{n+1} C_m(t)\right] - \left[\frac{C-S}{n} + \frac{1}{n}\sum_{m=1}^{n} C_m(t)\right]$$
$$= \left(\frac{C-S}{n+1}\right) + \frac{1}{n+1}\left[\sum_{m=1}^{n} C_m(t) + C_{n+1}(t)\right] \text{ —— } \cancel{(11)}$$
$$= \frac{-\left\{(C-S) + \sum_{m=1}^{n} C_m(t)\right\}}{n(n+1)} + \frac{1}{n+1} C_{n+1}(t) \quad \text{—— (11)}$$

~~n=n-1.~~

~~Put $\Delta A(n-1)$)~~

Put $n = n-1$.

$$\Delta A(n-1) = \frac{(C-S) + \sum_{m=1}^{n-1} C_m(t)}{(n-1)(n+1)} + \frac{1}{\cancel{n+1}\,n} C_n(t) \quad \text{—— (3)}$$

Using (a) condition.

i.e $\Delta A(n) > 0$ and $\Delta A(n-1) < 0$.

$$= \frac{-A(n)}{\cancel{n+1}} + \frac{1}{\cancel{n+1}} C_{n+1}(t) > 0.$$

$$\text{and} \quad \frac{-A(n-1)}{\cancel{n}} + \frac{1}{\cancel{n}} C_n(t) < 0.$$

or $C_{n+1}(t) > A(n)$ —— (b).

and $C_n(t) < A(n-1)$ —— (c).

by (a) maintance cost is greater than ~~(b)~~ Average ~~maintance cost~~ cost of next year.

then old machine is replace by new machine.

~~If next year maintance cost~~

By (c) if maintance cost is less than the Average cost of previous year then do not replace old machine by new machine.

Exercise :

Q10. To find the avrage cost per year of the machine.

given $C = 12200$ $S = 200$.

$C - S = 12000$. (Depreciation).

Year (1)	Depreciation (C-S) (2)	Maintance cost $C_m(t)$ (3)	Total maintance cost $\sum_{m=1}^{8} C_m(t)$ (4)	Total cost $(C-S) + \sum_{m=1}^{8} C_m(t)$. (5) = (2) + (4)	Average cost $A(n) = \frac{(C-S) + \sum_{m=1}^{n} C_m(t)}{n}$ (6) = (5)/n [n = 1, 2, 3, 4, 5, 6, 7, 8]
1	12000	200	200	12,200	12,200
2	12000	500	700	12,700	6,350
3	12000	800	1500	13,500	4500
4	12000	1200	2700	14,700	3675
5	12000	1800	4500	16,500	3300
6	12000	2500	7000	19,000	3166.6
7	12000	3200	10,200	22,200	3171.4
8	12000	4000	14,200	26,200	3275

since Avg. cost decrea[...] and

Increasing in 7th year then old machine must be

repalace at the end of six year or in the begi[...]

beginning

Few Important Terms.

(1) Money Value : The money obtained by inte[...] and many other way by amount i[...] called the money value.

If we borrow ₹100 at 10%. interest Then w[...] to pay ₹110 in place of 100.

The cost of ₹1 is $\frac{100}{110}$ after one year =[...]

similarly the cost of ₹1 after n year = (1.[...]

(2) Present value (or Present worth) :

The cost of 1₹ after n year is called th[...] present value.

(3) Discount Rate (or Depreciation Ratio):

The value of the money decreases with [...] constant ratio which is known as discou[...] rate or depreciation ratio.

It is denoted by 'v' and

$$\boxed{v = \frac{1}{1+r}}$$

where r is t[...] of interes[...]

4.5; The Best Replacement Age of Items whose maintenance Costs Increase with Time and the value of money also changes with Time.

$$\frac{1-v^{n-1}}{1-v} C_n - P(n-1) < 0 < \frac{1-v^n}{1-v} \cdot C_{n+1} - P(n).$$

Where

$$C(n) = \frac{A + \sum_{r=1}^{n} C_r v^{r-1}}{1-v^n} = \frac{P(n)}{1-v^n}.$$

A is cost of Machine.

P(n) is total expenditure on machine in 'n' years.

Example 10:

Sol^n: $A = ₹5000$; $C_n = 500(n-1)$, $n = 1, 2, 3, \dots$

Year	C_n ₹	$v^{n-1} = (0.5)^{n-1}$	$v^{n-1} \cdot C_n$	$(1-v^{n-1}) \cdot C_n$	$\frac{1-v^n}{1-v} \cdot C_{n+1}$	P(n)	$\frac{1-v^n}{1-v} \cdot C_{n+1} - P(n)$
(1)	(2)	(3)	(4)=(2)×(3)	(5)=(2)-(4)	(6)	(7)	(8)=(6)-(7)
1	0	1.0000	00.00	00.00	500.00	5000.00	< 0
2	500	0.5000	250.00	250.00	1500.00	5250.00	< 0
3	1000	0.2500	250.00	750.00	2625.00	5500.00	< 0
4	1500	0.1250	187.50	1312.50	3750.00	5687.50	< 0
5	2000	0.0625	125.00	1875.00	4844.00	5812.50	< 0
6	2500	0.0312	78.00	2422.00	5906.40	5890.50	> 0
7	3000	0.0156	46.80	2953.20	—	—	

From last table the average cost of machine decrease upto 5^{th} year and increase in 6 year. then old machine can be replaced by new new machine after 6 years.

In the end of 5^{th} year and beginning of 6^{th} year.

It is clear from the table that for $n=6$.

$$\frac{1-v^5}{1-v} C_6 - P(5) = ₹4844.00 - ₹5812.50$$

$$= -₹968.50 < 0 < \frac{1-v^6}{1-v} C_7 - P(6)$$

$$= ₹5906.40 - ₹5890.50$$

$\Rightarrow$ ₹ 15.90.

Hence, it will be economical to replace the machine by a new one after every six year.

Replacement of items when system is completely failed.

In such type of replacement policy all system must be replaced after the fixed time interval whether the system is failed or not there are two policies.

(1) Individual replacement policy : In individual replacement policy particular item must be replace b/w the fixed time interval.

(2) Group Replacement policy; In such type of policy include the cost of individual item and cost of group replacement item.

Let N = Total number of item in the system.

$N(x)$ = total number of item failed during the x^{th} period.

C_g = The cost per item when all the items are replaced in a group.

C_i is the cost of replacing when item failed.

$C(n)$ = Total cost in the interval t (including the costs of replacing individual at the end of each period) consisting of n peri

$= N \cdot C_g + C_i$ (Total number of failures in the period 1, 2, 3, ... (n - 1))

$= N \cdot C_g + C_i [N(1) + N(2) + N(3) + \dots + N(n-1)]$

or $C(n) = N \cdot C_g + C_i \sum_{x=1}^{n-1} N(x)$

$\therefore$ Average cost per period is given by

$A(n) = \frac{C(n)}{n}$.

$A(n)$ is mininum for that value of n, for which

$\Delta A(n-1) < 0 < \Delta A(n)$.

Now $\Delta A(n) = A(n+1) - A(n)$

$= \frac{C(n+1)}{n+1} - \frac{C(n)}{n} = \frac{C(n) + C_i N(n)}{n+1} - \frac{C(n)}{n}$.

$= \frac{n C_i N(n) - C(n)}{n(n+1)} = \frac{C_i \cdot N(n) - \frac{C(n)}{n}}{n+1}$.

Similarly $\Delta A(n-1) = \frac{C_i \cdot N(n-1) - \frac{C(n-1)}{n-1}}{n}$

$$\frac{C_i N(n-1) - \frac{C(n-1)}{n-1}}{n} < 0 < \frac{C_i N(n) - \frac{C(n)}{n}}{n+1}.$$

$$C_i N(n-1) - \frac{C(n-1)}{(n-1)} < 0 < C_i N(n) - \frac{C(n)}{n}$$

$C_i N(n) > \frac{C(n)}{n}$ (by the end of n year)

~~and~~ $\cancel{C_i N(n-1) < \frac{C(n-1)}{n-1}}$.

Individual cost is > average cost then old system must be replace by the new system.

$$C_i N(n-1) < \frac{C(n-1)}{n-1}.$$

do not replace when individual cost of (n-1) year is less than the average cost of (n-1) year.